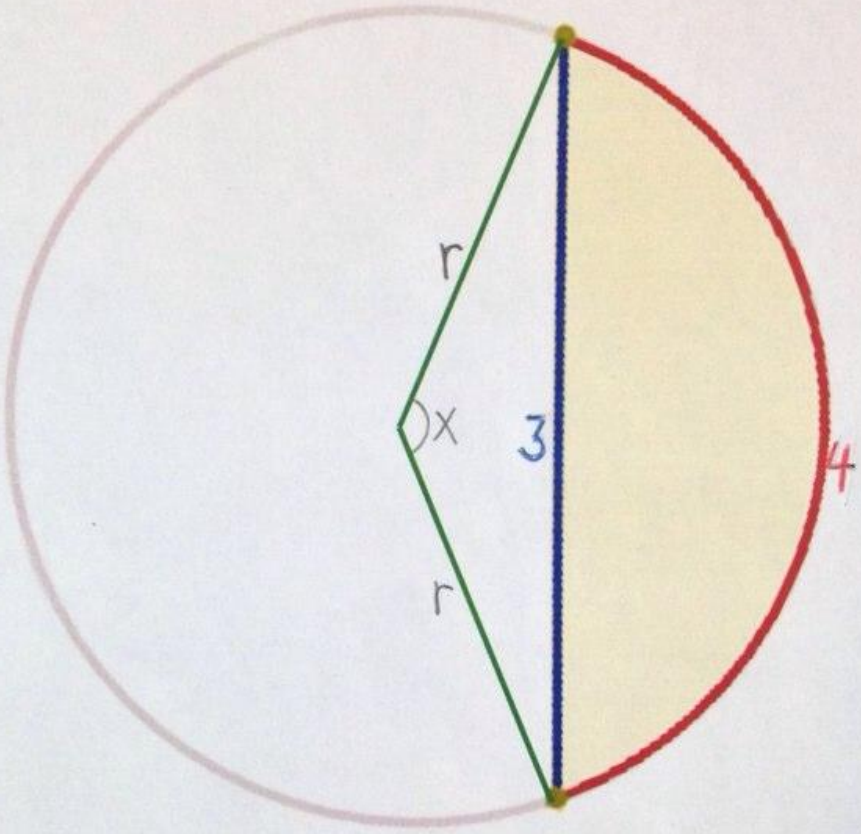
★**9.** Määritä segmentin ala A, kun sitä vastaava jänne on 3 ja sektorin kaaren pituus 4. Esitä vastaus kahden desimaalin tarkkuudella.



Keskuskolmio kosinilauseella:

$3^2 = r^2 + r^2 - 2 \cdot r \cdot r \cdot \cos x$

$9 = 2r^2 - 2r^2 \cdot \cos x$

$9 = 2r^2 \cdot (1 - \cos x) \quad \left| r = \frac{4}{x} \right.$

$9 = 2 \cdot \left(\frac{4}{x}\right)^2 \cdot (1 - \cos x)$

$9 = 2 \cdot \frac{16}{x^2} \cdot (1 - \cos x) \quad \| \cdot x^2$

Sektorin kaari:

$4 = \frac{x}{\frac{360°}{2\pi}} \cdot 2\pi r$

$4 = \frac{x}{2\pi} \cdot 2\pi r$

$4 = x \cdot r$

$r = \frac{4}{x}$

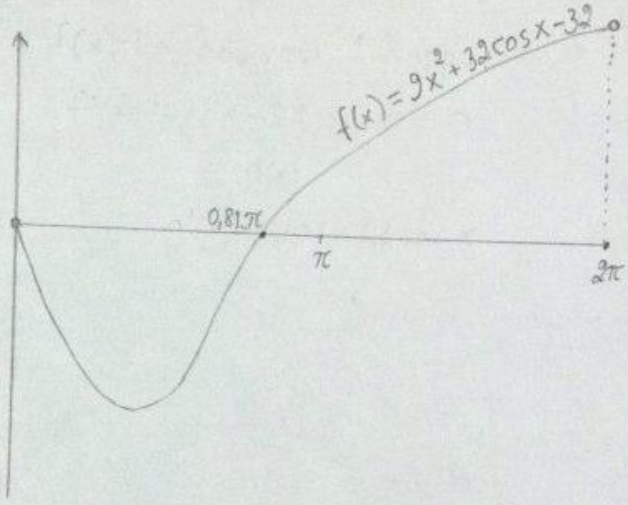
$9x^2 = 32 \cdot (1 - \cos x) \Rightarrow 9x^2 + 32\cos x - 32 = 0 \qquad x \in]0, 2\pi[$

(Ratkaistaan Newtonin menetelmällä)

$$f(x) = 9x^2 + 32\cos x - 32$$
$$f'(x) = 18x - 32\sin x$$

$$\boxed{X_{n+1} = X_n - \frac{(9X_n^2 + 32\cos X_n - 32)}{(18X_n - 32\sin X_n)}}$$

$f(2) < 0$
$f(3) > 0$ $\Rightarrow$ siemenluku $X_0 = 2{,}5$ $(= 0{,}795..\pi)$



$X_1 = 2{,}553...$
$X_2 = 2{,}55140...$
$X_3 = 2{,}551396218..$
$X_4 = 2{,}551396218..$

$X \approx 2{,}55139622$
$X = 0{,}81213464\pi$
$(\Rightarrow r = \frac{4}{X} = 1{,}5677...)$

Segmentin ala $A = \underbrace{\frac{X}{2\pi} \cdot \pi r^2}_{\text{sektori}} - \underbrace{\frac{1}{2} \cdot r \cdot r \cdot \sin x}_{\text{keskuskolmio}}$

$$A = \frac{X}{2} \cdot r^2 - \frac{1}{2} r^2 \cdot \sin x \quad \Big| r = \frac{4}{X}$$

$$A = \frac{X}{2} \cdot \left(\frac{4}{X}\right)^2 - \frac{1}{2} \cdot \left(\frac{4}{X}\right)^2 \cdot \sin x$$

$$A = \frac{8}{X} - \frac{8}{X^2} \cdot \sin x \quad \Big| X = 0{,}81213464\pi$$

$$A = 2{,}45159... \approx 2{,}452$$

$$\boxed{\text{Vastaus: } A = 2{,}452}$$